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The strong CP problem versus Planck scale physics

Bogdan A. Dobrescu[1]

Department of Physics, Boston University
590 Commonwealth Avenue, Boston, MA 02215, USA

Abstract

We discuss conditions that should be satisfied by axion models for solving the strong CP problem. It has been observed that Planck scale effects may render the axion models ineffective if there are gauge invariant operators of dimension less than 10 which break explicitly the Peccei-Quinn (PQ) symmetry. We argue that only those operators formed of fields which have vacuum expectation values are dangerous. Supersymmetric axion models fail to prevent even this restricted class of operators. Furthermore, the models that relate the PQ scale and the supersymmetry breaking scale are particularly sensitive to gauge invariant PQ-breaking operators. By contrast, in non-supersymmetric composite axion models the PQ scale arises naturally, and the dangerous operators can be avoided. However, the composite axion models contain heavy stable particles which are cosmologically ruled out. Another problem is a Landau pole for the QCD coupling constant. Both these problems may be solved if the unification of color with the gauge interactions which bind the axion could be achieved.

[1] e-mail address: dobrescu@budoe.bu.edu

1 Introduction

The fine tuning required to accommodate the observed CP invariance of the strong interactions, known as the strong CP problem [1], suggests that the strong CP parameter $\bar{\theta}$ is a dynamical field. If some colored fields are charged under a spontaneously broken Peccei-Quinn (PQ) symmetry [2], then $\bar{\theta}$ is replaced by a shifted axion field. The PQ symmetry is explicitly broken by QCD instantons, so that a potential for the axion is generated with a minimum at $\bar{\theta} = 0$. In the low energy theory, besides solving the strong CP problem, this mechanism predicts nonderivative couplings of the axion to the gauge bosons and model dependent derivative couplings of the axion to hadrons and leptons [3, 4].

There are two important issues that have to be addressed by axion models. First, Planck scale effects may break explicitly the PQ symmetry, shifting $\bar{\theta}$ from the origin [5, 6, 7]. Since only the gauge symmetries are expected to be preserved by Planck scale physics [8], the PQ symmetry should be a consequence of a gauge symmetry.

Second, an axion model should produce naturally the PQ symmetry breaking scale, $f_{\rm PQ}$. Astrophysics and cosmology [9] constrain the axion mass to lie between 10^{-5} and 10^{-3} eV [10], which translates in a range $10^{10} - 10^{12}$ GeV for $f_{\rm PQ}$. The small ratio between the PQ scale and the Planck scale, $M_{\rm P} \sim 10^{19}$ GeV, can be naturally explained if the PQ symmetry is broken dynamically in a theory with only fermions and gauge fields [11, 12]. Alternatively, if the PQ symmetry is broken by the vacuum expectation value (vev) of a fundamental scalar, then supersymmetry (susy) is required to protect $f_{\rm PQ}$ against quadratic divergences.

In this paper we study phenomenological constraints on axion models and point out potential problems of the models constructed so far. In section 2 we discuss under what conditions a gauge symmetry can protect $\bar{\theta}$ from Planck scale effects. We also list theoretical and phenomenological requirements that should be imposed on axion models. These conditions are illustrated in the case of non-supersymmetric composite axion models in section 3. In section 4 it is shown that previous attempts of preventing harmful PQ breaking operators in supersymmetric theories have failed. A discussion of the PQ scale in supersymmetric models is also included. Conclusions and a summary of results are presented in section 5.

2 Constraints on axion models

2.1 Protecting the axion against Planck scale effects

Gravitational interactions are expected to break any continuous or discrete global symmetries [8], so that gauge invariant nonrenormalizable operators suppressed by powers of $M_{\rm P}$ are likely to have coefficients of order one. In refs. [6, 7] it is argued that, under these circumstances, a solution to the strong CP problem requires any gauge invariant operator of dimension less than 10 to preserve the PQ symmetry. The reason is that the PQ-breaking operators change the potential for the axion such that the minimum moves away from $\bar{\theta} = 0$. However, this condition can be relaxed. If a PQ-breaking operator involves fields which do not have vevs, then its effect is an interaction of the axion with these fields. The exchange of these fields will lead to a potential for the axion which is suppressed by at least as many powers of $M_{\rm P}$ as the lowest dimensional PQ-breaking operator formed by fields which have vevs. Therefore, a natural solution to the strong CP problem requires that *gauge symmetries forbid any PQ-breaking operator of dimension less than 10 involving only fields which acquire vevs of order* $f_{\rm PQ}$.

This relaxed form is still strong enough to raise the question of whether we should worry that much about Planck scale effects which are mostly unknown. Furthermore, in ref. [13] it is argued that although the idea of wormhole-induced global symmetry breaking is robust, some modifications of the theory of gravity at a scale of $10^{-1}M_{\rm P}$ or topological effects in string theory could lead to exponentially suppressed coefficients of the dangerous operators. There are also arguments that strongly coupled heterotic string theory may contain PQ symmetries which are adequately preserved [14].

Nevertheless, since the theory of quantum gravity still eludes us, assigning exponentially small coefficients to all the gauge invariant PQ-breaking operators in the low energy theory can be seen as a worse fine-tuning than setting $\bar{\theta} < 10^{-9}$. To show this consider a scalar Φ, charged under a global $\mathrm{U}(1)_{\rm PQ}$ which has a QCD anomaly, with a vev equal to $f_{\rm PQ}$, and a dimension-k gauge invariant operator

$$\frac{c}{k!}\frac{1}{M_{\rm P}^{k-4}}\Phi^k \ , \tag{2.1}$$

where c is a dimensionless coefficient. Solving the strong CP problem requires

$$\frac{c}{k!}\frac{f_{\rm PQ}^k}{M_{\rm P}^{k-4}} < \bar{\theta}\, M_\pi^2 f_\pi^2 \ . \tag{2.2}$$

Here M_π is the pion mass, and $f_\pi \approx 93$ MeV is the pion decay constant. Therefore, the condition on c is

$$|c| \lesssim \bar{\theta}\, k!\, 10^{8(k-10)} \left(\frac{10^{11}\ \mathrm{GeV}}{f_{\mathrm{PQ}}}\right)^k , \tag{2.3}$$

which means that c is less finely tuned than $\bar{\theta}$ only if $k \geq 9$ ($k \geq 11$) for $f_{\mathrm{PQ}} = 10^{10}$ GeV ($f_{\mathrm{PQ}} = 10^{12}$ GeV).

2.2 General conditions

Even in its relaxed form, the condition of avoiding Planck scale effects is hard to satisfy simultaneously with the other requirements of particle physics, cosmology and astrophysics. In the remainder of this section we list some of the important issues in axion model building.

i) Gauge anomaly cancellation.

ii) The colored fields carrying PQ charges should not acquire vevs which break $SU(3)_C$ color.

iii) The stability of f_{PQ} requires either susy or the absence of fundamental scalars at this scale. Furthermore, any mass parameter except M_{P} should arise from the dynamics. Otherwise, fine-tuning the ratio $f_{\mathrm{PQ}}/M_{\mathrm{P}}$ is as troublesome as imposing $\bar{\theta} < 10^{-9}$, and the motivation for axion models is lost. Note that the usual DFSZ [15] and KSVZ [16] models do not satisfy this condition.

iv) The strong coupling constant should remain small above f_{PQ}, until M_{P} or some grand unification scale. The one-loop renormalization group evolution for the strong coupling constant, starting with 5 flavors from $\alpha_s(M_Z) = 0.115$, then at 175 GeV including the top quark, gives

$$\frac{1}{\alpha_s(f_{\mathrm{PQ}})} \approx 32.0 + \frac{7}{2\pi} \log\left(\frac{f_{\mathrm{PQ}}}{10^{11}\ \mathrm{GeV}}\right) . \tag{2.4}$$

Running then α_s from f_{PQ} to M_{P} gives $\alpha_s(M_{\mathrm{P}}) < 1$ if the coefficient of the β function is $b_0 \lesssim 10.6$. This corresponds to a maximum of 26 new flavors. In supersymmetric theories the above computation gives $\alpha_s(f_{\mathrm{PQ}}) \approx 1/19$, and $\alpha_s(M_{\mathrm{P}}) < 1$ if $b_0 \lesssim 6$, i.e. there can be at most 10 new flavors with masses of order f_{PQ}. If there are additional flavors below f_{PQ}, the total number of flavors allowed is reduced. In the case of composite axion models there are non-perturbative effects, due to the fields carrying the confining gauge interactions and the usual color, which change the running of α_s at scales close to f_{PQ} and can be only roughly estimated.

v) Composite stable particles with masses $M_{\rm comp}$ larger than about 10^5 GeV lead too early to a matter dominated universe [17]. It is then necessary that all stable particles with masses of order $f_{\rm PQ}$ to be short lived. Their energy density remains smaller than the critical one provided their lifetime is smaller than of order 10^{-8} seconds [18]. However, if there is inflation any unwanted relic is wiped out, and if the reheating temperature is lower than $f_{\rm PQ}$, then the heavy stable particles are not produced again and the above condition is no longer necessary.

vi) Domain walls may arise in many axion models [19], and they should disappear before dominating the matter density of the universe. Inflation takes care of this requirement too, but there are also other mechanisms for allowing the domain walls to evaporate [1, 7, 20].

vii) Any new colored particle should be heavier than about the electroweak scale [10]. Etc.

3 Composite axion

The PQ scale is about 9 orders of magnitude smaller than the Planck scale, which is unnatural unless the spontaneous breaking of the PQ symmetry is a consequence of non-perturbative effects of some non-Abelian gauge symmetry. In this section we concentrate on non-supersymmetric theories, and therefore we do not allow light (compared to $M_{\rm P}$) fundamental scalars. We have to consider then theories with fermions transforming non-trivially under a gauge group. From QCD it is known that the strong dynamics break the chiral symmetry of the quarks. Thus, if the PQ symmetry is a subgroup of a chiral symmetry in a QCD-like theory, then $f_{\rm PQ}$ will be of the order of the scale where the gauge interactions become strong. As a result the axion will be a composite state, formed of the fermions charged under the confining gauge interactions.

3.1 Kim's model

The idea of a composite axion is explicitly realized in the model presented in ref. [11], which contains fermions carrying color and the charges of an SU(N) gauge interaction, called axi-color. The left-handed fermions are in the following representations of the SU(N)×SU(3)$_C$ gauge group:

$$\psi : (N, 3)\,,\ \phi : (N, 1)\,,\ \chi : (\overline{N}, \bar{3})\,,\ \omega : (\overline{N}, 1)\ . \tag{3.1}$$

SU(N) becomes strong at a scale $\Lambda_{\rm a}$ of order $f_{\rm PQ}$ and the fermions condense. This is a QCD-like theory with N axicolors and 4 flavors, and from QCD we know that the condensates will preserve SU(N). In the limit where the $\mathrm{SU}(3)_C$ coupling constant, α_s, is zero, the channels of condensation which preserve color are equally attractive as the ones which break color. Thus, although α_s is small at the scale $\Lambda_{\rm a}$, its non-zero value will force the condensates to preserve color, which implies that only the $\langle\psi\chi\rangle$ and $\langle\phi\omega\rangle$ condensates will form.

In the limit $\alpha_s \to 0$, Kim's model has an $\mathrm{SU}(4)_L\times\mathrm{SU}(4)_{\overline{R}}\times\mathrm{U}(1)_{\rm V}$ global symmetry which is spontaneously broken down to $\mathrm{SU}(4)_{L-\overline{R}}\times\mathrm{U}(1)_{\rm V}$ by the condensates. The resulting 15 Goldstone bosons transform as $1+3+\bar{3}+8$ under $\mathrm{SU}(3)_C$. The color singlet is the composite axion, with a $(\psi\chi - 3\phi\omega)$ content, and $\mathrm{U}(1)_{\rm PQ}$ corresponds to the $[(Q_{\rm PQ})_L \times 1_{\overline{R}} + 1_L \times (Q_{\rm PQ})_{\overline{R}}]/\sqrt{2}$ broken generator of $\mathrm{SU}(4)_L\times\mathrm{SU}(4)_{\overline{R}}$, where

$$Q_{\rm PQ} = \frac{1}{2\sqrt{6}}\mathrm{diag}(1,1,1,-3) \ . \tag{3.2}$$

When α_s is turned on, the $\mathrm{SU}(4)_{L-\overline{R}}$ global symmetry is explicitly broken down to the gauged $\mathrm{SU}(3)_C$ and the global $\mathrm{U}(1)_{{\rm axi}-B-L}$ generated by $(Q_{\rm PQ})_{L-\overline{R}}$. The axion gets a tiny mass from QCD instantons, while the other (pseudo) Goldstone bosons get masses from gluon exchange.

Although the normalization of the PQ symmetry breaking scale, $f_{\rm PQ}$, is ambiguous, the axion mass, $m_{\rm a}$, is non-ambiguously related to the axicolor scale, $\Lambda_{\rm a}$, because $\mathrm{U}(1)_{\rm PQ}$ is a subgroup of the chiral symmetry of axicolor. To find this relation note first that the axion mass is determined by the "axi-pion" decay constant, f_a (the analog of f_π from QCD), by [4]

$$m_{\rm a} = \frac{4A^C_{\rm PQ}}{f_{\rm a}} M_\pi f_\pi \frac{Z^{1/2}}{1+Z} \ , \tag{3.3}$$

where $Z \approx 0.5$ is the up to down quark mass ratio, and $A^C_{\rm PQ}$ is the color anomaly of $\mathrm{U}(1)_{\rm PQ}$:

$$\delta_{ab} A^C_{\rm PQ} = N\,\mathrm{Tr}(T_a T_b Q_{\rm PQ}) \ . \tag{3.4}$$

The normalization of the $\mathrm{SU}(3)_C$ generators [embedded in $\mathrm{SU}(4)_{L-\overline{R}}$] is $\mathrm{Tr}(T_aT_b) = \delta_{ab}/2$, and we find

$$f_{\rm a} = 2.4\times 10^9\,\mathrm{GeV}\left(\frac{10^{-3}\,\mathrm{eV}}{m_{\rm a}}\right) N \ . \tag{3.5}$$

In the large-N limit the relation between $f_{\rm a}$ and $\Lambda_{\rm a}$ is

$$\frac{\Lambda_{\rm a}}{\Lambda_{\rm QCD}} = \frac{f_{\rm a}}{f_\pi}\sqrt{\frac{3}{N}} \ , \tag{3.6}$$

where $\Lambda_{\rm QCD} \sim 200$ MeV.

This model suffers from the energy density problem of stable composite particles [21] [see point v) in section 2]. The reason is that the global $\mathrm{U}(1)_{\rm V}$ (the analog of the baryon number symmetry in QCD) is an exact symmetry such that the lightest axibaryon is stable. Its mass is larger than $f_{\rm a}$ and can be evaluated as in ref. [22] by scaling from QCD:

$$M_{\rm aB} = m_p \left(\frac{f_{\rm a}}{f_\pi}\right) \sqrt{\frac{N}{3}}\ , \tag{3.7}$$

where m_p is the proton mass. If axicolor can be unified with a standard model gauge group, then the heavy gauge bosons would mediate the decay of the axibaryons into standard model fermions and the model would be cosmologically safe [18]. However, it will be highly non-trivial to achieve such a unification. The only attempt so far of avoiding the axibaryon cosmological problem involves scalars [21], so it is unsatisfactory unless one shows that these scalars can be composite states.

We point out that the axibaryons are not the only heavy stable particles: the color triplet pseudo Goldstone bosons (PGB's) have also a too large energy density. Their masses can be estimated by scaling the contribution from electromagnetic interactions to the pion mass, which is related to the difference between the squared masses of $\pi^\pm$ and π^0. Since $\alpha_s(\Lambda_{\rm a})$ is small, the bulk of the colored PGB's masses comes from one gluon exchange [23]:

$$M^2_{(R)} \approx C^2(R) \frac{\alpha_s(\Lambda_{\rm a})}{\alpha(\Lambda_{\rm QCD})} \frac{\Lambda_{\rm a}^2}{\Lambda^2_{\rm QCD}} \left(M^2_{\pi^\pm} - M^2_{\pi^0}\right)\ . \tag{3.8}$$

Here[2] R is the $\mathrm{SU}(3)_C$ representation, $C^2(R)$ is the quadratic Casimir, equal to 3 for the color octet and 4/3 for the triplet, and α is the electromagnetic coupling constant. Therefore, the color triplet PGB's , which are $\psi\omega$ and $\phi\chi$ bound states, have a mass

$$M_{(3,\bar{3})} \approx 0.9 f_{\rm a} \sqrt{\frac{3}{N}} \tag{3.9}$$

and, except for the axion, are the lightest "axihadrons". These are absolutely stable due to the exact global $\mathrm{U}(1)_{{\rm axi}-(B-L)}$ symmetry. One may choose though not to worry about stable axihadrons by assuming a period of inflation with reheating temperature below the PGB mass.

The model discussed so far does not attempt to avoid the Planck scale induced operators which violate the PQ symmetry. In fact, Kim's model is vector-like: the ψ and χ, as

[2]Eq. (3.8) improves the estimate given in [21, 24] by eliminating the dependence on N shown in eq. (3.6).

well as the ϕ and ω, will pair to form Dirac fermions. Their mass is likely to be of order $M_{\rm P}$ and then fermion condensation does not take place and the model becomes useless. Even if Planck scale masses for the fermions are not generated, there are dimension 6 operators which violate $\mathrm{U(1)_{PQ}}$:

$$\frac{c_1}{M_{\rm P}^2}(\psi\chi)^2 \ \ , \ \ \frac{c_2}{M_{\rm P}^2}(\phi\omega)^2 \ , \tag{3.10}$$

where c_j, $j = 1, 2$, are dimensionless coefficients. These operators will shift the vev of the axion such that $\bar{\theta}$ will remain within the experimental bound only if

$$\frac{9|c_1| + |c_2|}{M_{\rm P}^2}\left(4\pi f_{\rm a}^3\sqrt{\frac{3}{N}}\right)^2 < 10^{-9} M_\pi^2 f_\pi^2 \ , \tag{3.11}$$

implying $|c_j| < \mathcal{O}(10^{-47})$. It is hard to accept this tiny number given that the motivation for studying axion models is to explain the small value $\bar{\theta} < 10^{-9}$.

3.2 Randall's model

There is only one axion model in the literature which does not involve scalars and avoids large Planck scale effects [12]. To achieve this, Randall's model includes another gauge interaction, which is weak, in addition to the confining axicolor. The left-handed fermions transform under the SU(N)×SU(m)×$\mathrm{SU(3)}_C$ gauge group as:

$$\psi : (N, m, 3)\, ,\ \phi_i : (N, \overline{m}, 1)\, ,\ \chi_j : (\overline{N}, 1, \bar{3})\, ,\ \omega_k : (\overline{N}, 1, 1)\ , \tag{3.12}$$

where $i = 1, 2, 3$, $j = 1, ..., m$, and $k = 1, ..., 3m$ are flavor indices. Axicolor SU(N) becomes strong at the $\Lambda_{\rm a}$ scale and the fermions condense. If the SU(m) gauge coupling, g_m, is turned off, the vacuum will align as in Kim's model and will preserve color. When g_m is non-zero, the SU(m) gauge interaction will tend to change the vacuum alignment and break the SU(N) gauge symmetry. However, since g_m is small, this will not happen, as we know from QCD where the weak interactions of the quarks do not affect the quark condensates. Therefore, the

$$\frac{1}{3}\langle\psi\chi_j\rangle = \langle\phi_i\omega_k\rangle \approx 4\pi f_{\rm a}^3\sqrt{\frac{3}{N}}\ . \tag{3.13}$$

condensates are produced, breaking the SU(m) gauge group and preserving color. A global $\mathrm{U(1)_{PQ}}$, under which ψ and χ_j have charge +1 while ϕ_i and ω_k have charge −1, is spontaneously broken by the condensates, so that an axion arises.

The lowest dimensional gauge invariant and PQ-breaking operators involving only fields that acquire vevs are

$$c^{ijk}_{m'} \frac{1}{M_{\rm P}^{3m-4}} \left(\psi\chi_j\right)^{m-m'} \left(\overline{\phi_i}\overline{\omega_k}\right)^{m'} \ , \tag{3.14}$$

with $m' = 1, ..., m$ ($m' \neq m/2$). The $c^{ijk}_{m'}$ coefficients are assumed to be of order one. The solution to the strong CP problem requires

$$\frac{C(m)}{M_{\rm P}^{3m-4}} \left(4\pi f_{\rm a}^3 \sqrt{\frac{3}{N}}\right)^m < 10^{-9} M_\pi^2 f_\pi^2 \ , \tag{3.15}$$

where

$$C(m) \equiv \sum_{ijkm'} 3^{m-m'} \left|c^{ijk}_{m'}\right| \ . \tag{3.16}$$

A necessary condition that follows from inequality (3.15) is $3m \geq 10$. Note that the window $f_{\rm PQ} \sim 10^7$ GeV discussed in [12] has been closed [25]. This constraint on m, combined with the condition of asymptotic freedom for SU(N) gives a lower bound for N,

$$\frac{11}{12} N > m \geq 4 \ . \tag{3.17}$$

We will see shortly that $m = 4$, $N = 5$ are the only values that may not lead to a Landau pole for QCD much below $M_{\rm P}$. For these values of m, inequality (3.15) yields an upper limit for the axi-pion decay constant:

$$f_{\rm a} < \frac{1.9 \times 10^{11}\,{\rm GeV}}{C(4)^{1/12}} \ . \tag{3.18}$$

For random values of order one of the $c^{ijk}_{m'}$ coefficients, we expect $C(4)^{1/12}$ to be between 1.5 and 2.5. For example, if $c^{ijk}_{m'} = 1$, then $C(m) = 9m^2(3^{m+1} - 1)/2$, which gives $C(4)^{1/12} \approx 2.25$. Therefore, $f_{\rm a} \lesssim 10^{11}$ GeV is necessary for avoiding fine-tuning of the higher dimensional operators in the low energy effective Lagrangian.

We note that $m = 4$ allows dimension-9 gauge invariant operators which break $\rm U(1)_{PQ}$:

$$(\overline{\phi_i}\psi)^2(\overline{\chi_j}\omega_k) \ , \ (\psi\omega_k)^2(\overline{\phi_i}\psi) \ , \ (\overline{\psi}\phi_i)(\phi_l\chi_j)^2 \ . \tag{3.19}$$

However, these are not harmful because they are not formed of the fields which acquire vevs, i.e. $(\psi\chi_j)$ and $(\phi_i\omega_k)$. They will just induce suppressed interactions of the axion with the fermions. Hence, this model is an example where the redundant condition of avoiding *all* the operators of dimension less than 10 is not satisfied.

Randall's model has a non-anomalous SU(3m)$\times$SU(m)$\times$SU(3)$\times$U$(1)_{\mathrm{axi}-(B-L)}$ global symmetry under which the fermions transform as:

$$\psi : (1, 1, 1)_{+1} \, , \ \phi : (1, 1, 3)_{-1} \, , \ \chi : (1, m, 1)_{-1} \, , \ \omega : (3m, 1, 1)_{+1} \ . \tag{3.20}$$

This global symmetry, combined with the SU(m) gauge symmetry, is spontaneously broken down to an [SU(m)$\times$SU(3)]$_{\mathrm{global}}\times$U$(1)_{\mathrm{axi}-(B-L)}$ global symmetry by the condensates. Thus, there are $10m^2 - 2 = 158$ Goldstone bosons: $m^2 - 1$ of them are eaten by the SU(m) gauge bosons which acquire a mass $g_m f_{\mathrm{PQ}}/2$, while the other $9m^2 - 1$ get very small masses from higher dimensional operators. These are color singlets, very weakly coupled to the standard model particles, and, as pointed out in [12], their energy density might not pose cosmological problems.

The Goldstone bosons have $\psi\chi$ and $\phi\omega$ content and transform in the $(m, 1)$ and $(m, 3)$ representations of the unbroken [SU(m)$\times$SU(3)]$_{\mathrm{global}}$, respectively. Therefore, these symmetries do not prevent heavy resonances from decaying into Goldstone bosons, and are cosmologically safe. However, as in Kim's model, the lightest particles carrying axi-$(B-L)$ number are the color triplet PGB's, which have $\psi\omega_k$ and $\phi_i\chi_j$ content and are heavy due to gluon exchange. Hence, there are $18m^2$ stable "aximesons" with masses given by eq. (3.9), which pose cosmological problems.

Besides heavy stable particles, there are meta-stable states, with very long lifetimes, incompatible with the thermal evolution of the early universe. To show this we observe that there is an axibaryon number symmetry, U$(1)_{\mathrm{V}}$, broken only by the SU(m) anomaly. The ψ and χ fermions have U$(1)_{\mathrm{V}}$ charge $+1$ while ϕ and ω have charge -1. The lightest axibaryons are the color singlet $\psi^{3p_1}\phi^{3p_2}\chi^{p_3}\omega^{p_4}$ states [22], with $p_l \geq 0$ $(l = 1, ..., 4)$ integers satisfying $\sum p_l = N$. These can decay at low temperature only via SU(m) instantons, with a rate proportional to $\exp(-16\pi^2/g_m^2)$, which is extremely small given that the SU(m) gauge coupling is small. At a temperature of order f_{PQ} transitions between vacua with different axibaryon number via sphalerons will affect the axibaryon energy density. Nonetheless, this thermal effect is exponentially suppressed as the universe cools down such that the order of magnitude of the axibaryon energy density is unlikely to have time to change significantly.

As in Kim's model, unification of axicolor with other gauge groups will allow axihadron decays if the axicolored fermions belong to the same multiplet as some light fermions. In this model though it seems even more difficult to unify axicolor with other groups. Inflation with reheating temperature below the axibaryon mass M_{aB} [see eq. (3.7)] appears

then a necessary ingredient.

Another problem may be the existence of a large number of colored particles. QCD not only loses asymptotic freedom but in fact the strong coupling constant may hit the Landau pole below $M_{\rm P}$. To study this issue we need to evaluate $\alpha_s(M_{\rm P})^{-1}$. Below the scale set by the mass of the PGB's, the effects of the "axihadrons" on the running of α_s are negligible and we can use eq. (2.4). Above some scale $\Lambda_{\rm pert}$ larger than $4\pi f_a/\sqrt{N}$ the perturbative renormalization group evolution can again be used, with $mN+6$ flavors. However, at scales between the mass of the PGB's and $\Lambda_{\rm pert}$, besides the perturbative contributions from the gluons and the six quark flavors, there are large non-perturbative effects of the axicolor interaction which are hard to estimate. We can write

$$\frac{1}{\alpha_s(M_{\rm P})} = 32.0 + \frac{7}{2\pi}\log\left(\frac{M_{\rm P}}{10^{11}\,{\rm GeV}}\right) - \frac{mN}{3\pi}\log\left(\frac{M_{\rm P}}{\Lambda_{\rm pert}}\right) - \delta_{\rm PGB} - \delta_{\rm axicolor}\ . \tag{3.21}$$

Here $\delta_{\rm PGB}$ is the contribution from colored PGB's, and $\delta_{\rm axicolor}$ is the non-perturbative contribution of the axicolored fermions, which can be interpreted as the effect of the axihadrons on the running of α_s. Axicolor interactions have an effect on the size of these two non-perturbative contributions, but it is unlikely that they change the signs of the one-loop contributions from PGB's and axihadrons. Therefore, we expect $\delta_{\rm PGB}$ and $\delta_{\rm axicolor}$ to be positive. This is confirmed by the estimate of the hadronic contributions to the photon vacuum polarization [26] within relativistic constituent quark models, and by the study of the running of α_s in technicolor theories [27], which indicate

$$\delta_{\rm axicolor} > \delta_{\rm PGB} > 0\ . \tag{3.22}$$

From eq. (3.21) one then can see that $\alpha_s^{-1}(M_{\rm P})$ is negative for any m and N larger than the smallest values allowed by eq. (3.17): $m=4$, $N=5$. With these values eq. (3.21) becomes

$$\frac{1}{\alpha_s(M_{\rm P})} = 13.4 + \frac{20}{3\pi}\log\left(\frac{\Lambda_{\rm pert}}{10^{11}\,{\rm GeV}}\right) - \delta_{\rm PGB} - \delta_{\rm axicolor}\ . \tag{3.23}$$

At low energies compared to $\Lambda_{\rm a}$, $\delta_{\rm PGB}$ can be evaluated using chiral perturbation theory [26]. Furthermore, as discussed in ref. [27] for the case of technicolor theories, the result can be estimated up to a factor of 2 by computing the one-loop PGB graphs. At energies larger than $\Lambda_{\rm a}$ chiral perturbation theory is not useful and the contribution to $\delta_{\rm PGB}$ is unknown. Keeping this important caveat in mind we will evaluate the one-loop PGB contributions. The leading log term from the $3m^2$ color triplet PGB's with mass

$M_{(3,\bar{3})}$ [see eq. (3.9)] and the m^2 color octet PGB's with mass $(9/4)M_{(3,\bar{3})}$ is given by

$$\delta_{\rm PGB} \approx K\frac{m^2}{\pi}\log\left(\sqrt{\frac{2}{3}}\frac{\Lambda_{\rm pert}}{M_{(3,\bar{3})}}\right) \ . \tag{3.24}$$

K is a constant between 1 and 2 which accounts for higher order corrections. Using eqs. (3.22)-(3.24) we can write

$$\frac{1}{\alpha_s(M_{\rm P})} < 11.8 - \frac{76}{3\pi}\log\left(\frac{\Lambda_{\rm pert}}{f_{\rm a}}\right) - \frac{20}{3\pi}\log\left(\frac{10^{11}\ {\rm GeV}}{f_{\rm a}}\right) \ , \tag{3.25}$$

where we used $K = 1$. The right-hand side of this inequality is negative because $f_a \lesssim 10^{11}$ GeV [see eq. (3.18)] and $\Lambda_{\rm pert}/f_a > 4\pi/\sqrt{5}$, which means that the strong coupling constant hits the Landau pole below $M_{\rm P}$.

Although the estimate of the non-perturbative effects on the RGE is debatable, this conclusion seems quite robust. A possible resolution would be to embed $SU(3)_C$ in a larger gauge group. In doing so, axicolor will lose asymptotic freedom unless it is also embedded in the larger group. Such an unification of color and axicolor would solve both problems discussed here: heavy stable particles and the Landau pole of QCD. However, it remains to be proved that this unification is feasible, given the large groups already involved.

4 Supersymmetric axion models

4.1 Planck scale effects in supersymmetric models

Apparently it is easier to build supersymmetric models in which the axion is protected against Planck scale effects because the holomorphy of the superpotential eliminates many of the higher dimensional operators. In practice, susy is broken so that the holomorphy does not ensure $\bar{\theta} < 10^{-9}$.

For example, consider the model presented in [6]. This is a GUT model with $E_6 \times U(1)_X$ gauge symmetry under which the chiral superfields transform as

$$\Phi :\ \overline{351}_0\ ,\quad \Psi_+ :\ 27_{+1}\ ,\quad \Psi_- :\ 27_{-1}\ . \tag{4.1}$$

The renormalizable superpotential,

$$W = \kappa\Phi\Psi_+\Psi_-\ , \tag{4.2}$$

has a $U(1)_{\rm PQ}$ under which Φ has charge -2 and Ψ_+, Ψ_- have charge $+1$. This is broken by dimension-6 and higher operators in the superpotential:

$$W_{\rm nr} = \frac{1}{M_{\rm P}^3}\left(\frac{\kappa_1}{6}\Phi^6 + \frac{\kappa_2}{3}(\Psi_+\Psi_-)^3 + \frac{\kappa_3}{4}\Phi^4\Psi_+\Psi_-\right) + ... \tag{4.3}$$

where the coefficients κ_j are expected to be of order one. As observed in ref. [28], their interference with the renormalizable superpotential gives dimension-7 operators in the Lagrangian:

$$\frac{1}{M_{\rm P}^3}\Phi^5\Psi_+^\dagger\Psi_-^\dagger \ , \quad \frac{1}{M_{\rm P}^3}\Psi_\pm^5\Psi_\mp^\dagger\Phi^\dagger \ , \quad \frac{1}{M_{\rm P}^3}\Phi^4\Phi^\dagger\left|\Psi_\pm\right|^2 \ , \tag{4.4}$$

where we use the same notation for the scalar components as for the corresponding chiral superfields. The only fields which acquire vevs are the scalar components of Φ (the Higgs). Therefore, according to the arguments of section 2.1, the operators (4.4) do not affect the solution to the strong CP problem because they involve the scalar components of Ψ_+ and Ψ_-, which have no vevs. The lowest dimensional operator in the supersymmetric Lagrangian formed only of the Φ scalars is $\Phi^{11}\Phi^{\dagger 5}$, and is given by the interference of the Φ^6 and Φ^{12} terms in the superpotential.

However, the situation changes when soft susy breaking terms are introduced. Consider the

$$\kappa' m_s \Phi\Psi_+\Psi_- \tag{4.5}$$

trilinear scalar term, where κ' is a dimensionless coupling constant, and m_s is the mass scale of susy breaking in the supersymmetric standard model. The exchange of a Ψ_+ and a Ψ_- scalar between this operator and the first operator in (4.4) leads at one loop to a six-scalar effective term in the Lagrangian:

$$-\frac{2\kappa^*\kappa_1\kappa'}{(4\pi)^2}\frac{m_s}{M_{\rm P}^3}\log\left(\frac{M_{\rm P}}{m_s}\right)\Phi^6 \ . \tag{4.6}$$

The constraint from $\bar\theta$ given by eq. (2.2) yields

$$|\kappa\kappa_1\kappa'| < \mathcal{O}(10^{-18}) \ , \tag{4.7}$$

where we have used $m_s \sim \mathcal{O}(250 \text{ GeV})$. Note that there are also one-loop $\Phi^6\left|\Phi\right|^{2n}$ terms which can be summed up. In addition, once soft susy breaking masses are introduced, the unwanted five-scalar term $\Phi^4\Phi^\dagger$ is induced at one-loop by contracting the Ψ legs of the third operator in (4.4). This term is independent of the trilinear soft term (4.5). Thus, the coupling constants that appear in the renormalizable superpotential, in the soft terms, or

in the non-renormalizable terms from the superpotential have to be very small, contrary to the goal of this model.

In ref. [28] it is suggested that an additional chiral superfield, Υ, which transforms non-trivially under both E_6 and $U(1)_X$, may allow different X charges for the Φ, Ψ_1 and Ψ_2 superfields while satisfying the gauge anomaly cancellation and avoiding the dangerous PQ breaking operators. We point out that Υ should transform in a real representation of E_6 to preserve the $(E_6)^3$ anomaly cancellation. The lowest real representation is the adjoint 78 and has index 2 in the normalization where the fundamental 27 and the antisymmetric 351 have indices 1/2 and 25/2, respectively. The gauge invariance of the renormalizable superpotential (4.2) requires $X_{\Psi_1} + X_{\Psi_2} = -X_\Phi$, which, together with the $(E_6)^2 \times U(1)_X$ anomaly cancellation gives $X_\Upsilon = -6X_\Phi$. Using these equations, we can write the $(U(1)_X)^3$ anomaly cancellation condition as a relation between the $U(1)_X$ charges of Ψ_1 and Ψ_2:

$$(X_{\Psi_1} + X_{\Psi_2})\left(X_{\Psi_1}^2 + \frac{407}{204}X_{\Psi_1}X_{\Psi_2} + X_{\Psi_2}^2\right) = 0 \ . \tag{4.8}$$

The only real solution of this equation is $X_{\Psi_1} = -X_{\Psi_2}$ which does not prevent the dangerous operator (4.6). The next real representation, 650, looks already too large to allow a viable phenomenology. Another proposal suggested in [28] assumes fermion condensation which now it is known not to occur in supersymmetric theories [29].

4.2 The problem of PQ symmetry breaking scale

If susy is relevant at the electroweak scale, and susy breaking is transmitted to the fields of the standard model by non-renormalizable interactions suppressed by powers of M_P, such as supergravity, then susy should be broken dynamically at a scale $M_S \sim 10^{11}$ GeV. This will give scalar masses of order M_W. A gauge singlet in the dynamical susy breaking (DSB) sector with a vev for the F-term of order M_S^2 would produce gaugino masses of order M_W [30]. However, any gauge singlet is likely to have a mass of order M_P, so that its vev would need to be highly fine-tuned. Nonetheless, gluino, neutralino and chargino masses of order M_W can be produced without need for gauge singlets if there are new non-Abelian gauge interactions which become strong at ~ 1 TeV [31].

The success of this scheme makes physics at the M_S scale an important candidate for spontaneously breaking a PQ-symmetry. More important, the existence of M_S in the rather narrow window allowed for f_{PQ} is worth further exploration. Nevertheless, models which break both susy and the PQ symmetry face serious challenges, which were not addressed in the past [32]. One obstacle is that the inclusion of colored fields in a model

of dynamical susy breaking typically results in a strongly coupled QCD right above M_S [30]. This problem can be solved by constructing a PQ sector that communicates with the DSB sector only through a weak gauge interaction in a manner analogous to the gauge mediated susy breaking models [33]. A more serious problem is the following: if colored superfields could be included in the DSB sector, they would have masses of order 10^{11} GeV and a non-supersymmetric spectrum. This will lead to large masses for the squarks, which in turn will destabilize the electroweak scale.

The troublesome colored fields from the DSB sector can be avoided if the fields of both the DSB sector and the visible sector transform under the same global $U(1)_{PQ}$, which is spontaneously broken in the DSB sector and explicitly broken in the visible sector by the color anomaly. As pointed out in ref. [34], this may be possible because the axion can be identified with one of the complex phases from the soft susy breaking terms of the supersymmetric standard model. However, it appears very difficult to protect the axion against Planck scale effects. For example, the μ and B terms break this $U(1)_{PQ}$ which means that they should be generated by the vevs of PQ-breaking products of fields from the DSB sector. These products of fields are gauge invariant and therefore Planck scale induced PQ-breaking operators may be induced. The naturalness of the axion solution is preserved provided these operators are suppressed by many powers of M_P, which in turn requires the vevs from the DSB sector to be much above M_S in order to generate large enough μ and B terms. Thus, this situation seems in contradiction with the cosmological bounds on the PQ scale. It should be mentioned though that a larger f_{PQ} might be allowed in certain unconventional cosmological scenarios suggested by string theory [35]. Note, however, that for larger f_{PQ} the constraints on PQ-breaking operators are significantly stronger [see eq. (2.2)].

Another possibility, discussed in refs. [36], is to relate f_{PQ} to the susy breaking scale from the visible sector. The idea is to induce negative squared masses at one-loop for some scalars, and to balance these soft susy breaking mass terms against some terms in the scalar potential coming from the superpotential which are suppressed by powers of M_P. By choosing the dimensionality of these terms, one can ensure that the minimum of the potential is in the range allowed for f_{PQ}. This mechanism is also very sensitive to Planck scale effects because it assumes the absence of certain gauge invariant PQ breaking operators of dimension one, two and three from the superpotential.

Given these difficulties in relating f_{PQ} to the susy breaking scale while avoiding the harmful low-dimensional operators, one may consider producing the PQ scale naturally by

introducing some gauge interactions which become strong at about 10^{11} GeV and break the PQ symmetry without breaking susy. Because this scenario is less constrained, it may be easier to avoid the PQ-breaking operators.

5 Conclusions

We have argued that an axion model has to satisfy two naturalness conditions in order to solve the strong CP problem:

a) the absence of low-dimensional Planck-scale induced PQ-breaking operators formed of fields which acquire vevs;

b) the absence of fundamental mass parameters much smaller than the Planck scale.

If these conditions are not satisfied, the models may not be ruled out given that the Planck scale physics is unknown, but the motivation for the axion models (i.e. avoiding fine-tuning) is lost.

Non-supersymmetric composite axion models satisfy condition b) easily. The only phenomenological problem is that they predict heavy stable particles which are ruled out by the thermal evolution of the early universe. However, this problem disappears if there is inflation with reheating temperature below the PQ scale. Condition a) is more troublesome. It is satisfied by only one composite axion model [12], and our estimate shows that it leads to a Landau pole for QCD. One may hope though that the uncertainty in the value of $M_{\rm P}$, i.e. the possibility of quantum gravitational effects somewhat below 10^{19} GeV, combined with unknown non-perturbative effects of axicolor on the running of the strong coupling constants, might push the Landau pole just above $M_{\rm P}$ where is irrelevant for field theory. But because this does not seem to be a probable scenario, it would be useful to study in detail the possibility of unifying color with axicolor.

By contrast, the existing supersymmetric models do not satisfy condition a). The models which attempt to eliminate the PQ-breaking operators rely on the holomorphy of the superpotential. We have shown that once susy breaking is taken into account, the PQ-breaking operators are reintroduced with sufficiently large coefficients (in the absence of fine-tuning) to spoil the solution to the strong CP problem. Also, the models that satisfy condition b) by relating the PQ scale to the susy breaking scale are particularly sensitive to gauge invariant PQ-breaking operators. These results suggest the need for further model building efforts.

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